



IIT-JEE / NEET / FOUNDATION (IX & X)

Time: 2 hours

Math | IIT-JEE

Marks: 50

(Straight line, Circles & Sequence serial)

NAME OF THE STUDENT:- _____

DATE:- _____

INSTRUCTION – ATTEMPT ALL QUESTIONS

- Q.1. For $a > b > c > 0$ the distance between (I.I) and the point of intersection of the lines $ax + by + c = 0$ and $bx + ay + c = 0$ is less than $\sqrt{2}$ then
- (a) $a + b - c > 0$ (b) $a - b + c < 0$ (c) $a - b + c > 0$ (d) $a + b - c > 0$
- Q.2. For a point p in the plane, let $d_1(p)$ and $d_2(p)$ be the distance of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 < d_1(p) + d_2(p) < 4$ is _____
- Q.3. The locus of the mid – point of the perpendiculars drawn from points on the line $x = 2y$ to the line $x = y$ is _____
- (a) $2x - 3y = 0$ (b) $5x - 7y = 0$ (c) $3x - 2y = 0$ (d) $7x - 5y = 0$
- Q.4. A straight line L at a distance of 4 units from the origin makes positive intercepts on the co – ordinate axes and the perpendicular from the origin to this line makes an angle of 60° with the line $x + y = 0$ then an equation of the line
- (a) $x + \sqrt{3}y = 8$ (b) $(\sqrt{3} + 1)x + (\sqrt{3} - 1)y = 8\sqrt{2}$
- (c) $\sqrt{3}x + y = 8$ (d) none of these
- Q.5. Find the value of m^2 for which the lines joining origin to the point of intersection of $y = mx - 1$ with $x^2 + 4xy + 3y^2 - 1 = 0$ are perpendicular to each other.

- Q.6. The number of integral value of K for which the line $3x + 4y = K$ intersects the circle $x^2 + y^2 - 2x - 4y + 4 = 0$ at two distinct points is _____
- Q.7. If $a > 2b > 0$ then the positive value of m for which $y = mx - b\sqrt{1 + m^2}$ is a common tangent to $x^2 + y^2 = b^2$ and $(x - a)^2 + y^2 = b^2$ is
- (a) $\frac{2b}{\sqrt{a^2 - 4b^2}}$ (b) $\frac{\sqrt{a^2 - 4b^2}}{2b}$ (c) $\frac{2b}{a - 2b}$ (d) $\frac{b}{a - 2b}$
- Q.8. Let the point B be the reflection of the point A (2,3) with respect to the line $8x - 6y - 23 = 0$. Let T_A and T_B be circles of radii 2 and 1 with centres A and B respectively. Let T be a common tangent to the circles T_A & T_B such that both the circles are on the same side of T. If c is the point of intersection of T and the line segment AC is _____
- Q.9. A circle is given by $x^2 + (y - 1)^2 = 1$ another circle c touches it externally and also the x-axis, then the locus of its centre is
- (a) $\{(x,y): x^2 = 4y\} \cup \{(x,y); y \leq 0\}$ (b) $\{(x,y): x^2 + (y-1)^2 = 4\} \cup \{(x,y); y \leq 0\}$
(c) $\{(x,y): x^2 = y\} \cup \{(0,y); y \leq 0\}$ (d) $\{(x,y): x^2 = 4y\} \cup \{(0,y); y \leq 0\}$
- Q.10. If one of the diameters of the circle given by the equation, $x^2 + y^2 + 4x + 6y - 12 = 0$: is a chord of a circle of radius 5, whose centre is at (-3,2) then the radius of the circle is _____?
- Q.11. If m arithmetic means (AMs) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4th term is equal to 2nd term then m is equal to _____
- Q.12. Let m be the minimum possible value of $\log_3(3)$
- Q.13. Let m be the minimum possible value of $\log_3(3^{y_1} + 3^{y_2} + 3^{y_3})$, where y_1, y_2, y_3 are real numbers for which $y_1 + y_2 + y_3 = 9$. Let M be the maximum possible value of $(\log_3 x_1 + \log_3 x_2 + \log_3 x_3)$, where x_1, x_2, x_3 are positive real number for which $x_1 + x_2 + x_3 = 9$. Then the value of $\log_2(m^3) + \log_3(M^2)$ is _____
- Q.14. A straight line through the vertex P of a triangle PQR intersects the side QR at the point S and circumcircle of the triangle PQR at the point T. If S is not the centre of circumcircle, then
- (a) $\frac{1}{PS} + \frac{1}{ST} < \frac{2}{\sqrt{QS \times SR}}$ (b) $\frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{QS \times SR}}$
(c) $\frac{1}{PS} + \frac{1}{ST} < \frac{4}{QR}$ (d) $\frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$
- Q.15. Let a_1, a_2, a_3, \dots be a sequence of positive integers in arithmetic progression with common difference 2. Also let b_1, b_2, b_3, \dots be a sequence of positive integers in geometric progression with common ratio 2. If $a_1 = b_1 = c$, then the number of all possible values of c,

for with equality

$$2(a_1 + a_2 + \dots + a_n) = b_1 + b_2 + \dots + b_n$$

Holds for some positive integer n, is _____

Q.16. Let a, b, c be positive integers such that $\frac{b}{a}$ is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is b + 2 the value of $\frac{a^2 + a - 14}{a + 1}$ is

Q.17. Let a, b, c, d be real number in G.P. If u, v, w, satisfy the system of equation

$$u + 2v + 3w = 6$$

$$4u + 5v + 6w = 12$$

$$6u + 9v = 4$$

then show that the roots of the equation

$$\left(\frac{1}{u} + \frac{1}{v} + \frac{1}{w}\right)x^2 + [(b-c)^2 + (c-a)^2 + (d-b)^2]x + u + v + w = 0$$

and $20x^2 + 10(a-d)^2x - 9 = 0$ are reciprocals of each other

Q.18. If $S_1, S_2, S_3, \dots, S_n$ are the sums of infinite geometric series whose first terms are 1, 2, 3, _____, n and whose common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{n+1}$ respectively, then find the value of $S_1^2 + S_2^2 + S_3^2 + \dots + S_{2n-1}^2$