Notes OF Mathematics for Class 12

Relations and Functions

Relation

If A and B are two non-empty sets, then a relation R from A to B is a subset of A x B.

If R ⊆ A x B and (a, b) ∈ R, then we say that a is related to b by the relation R, written as aRb

Domain and Range of a Relation

Let R be a relation from a set A to set B. Then, set of all first components or coordinates of the

Ordered pairs belonging to R is called: the domain of R, while the set of all second components

Or co-ordinates = of the ordered pairs belonging to R is called the range of R.

Thus, domain of R = {a: (a, b) ∈ R} and range of R = {b: (a, b) ∈ R}

Types of Relation

(i) Void Relation As Φ ⊂ A x A, for any set A, so Φ is a relation on A, called the empty or

Void relation.

(ii) Universal Relation Since, A x A ⊆ A x A, so A x A is a relation on A, called the universal

Relation.

(iii) Identity Relation The relation IA = {(a, a): a ∈ A} is called the identity relation on A.

(iv) Reflexive Relation A relation R is said to be reflexive relation, if every element of A is

Related to itself.

Thus, (a, a) ∈ R, ∀ a ∈ A = R is reflexive.

(v) Symmetric Relation A relation R is said to be symmetric relation, if

(a, b) ∈ R (b, a) ∈ R, ∀ a, b ∈ A

i.e., a R b ⇒ b R a, ∀ a, b ∈ A

⇒ R is symmetric.

(vi) Anti-Symmetric Relation A relation R is said to be anti-symmetric relation, iff.

(a, b) ∈ R and (b, a) ∈ R ⇒ a = b, ∀ a, b ∈ A

(vii) Transitive Relation A relation R is said to be transitive relation, iff (a, b) ∈ R and (b, c) ∈ R

⇒ (a, c) ∈ R, ∀ a, b, c ∈ A

(viii) Equivalence Relation A relation R is said to be an equivalence relation, if it is Simultaneously reflexive, symmetric and transitive on A.

(ix) Partial Order Relation A relation R is said to be a partial order relation, if it is Simultaneously reflexive, symmetric and anti-symmetric on A.

(x) Total Order Relation A relation R on a set A is said to be a total order relation on A, if R is a partial order relation on A.

Inverse Relation

If A and B are two non-empty sets and R be a relation from A to B, such that R = {(a, b) : a ∈ A, b ∈ B}, then the inverse of R, denoted by R^{-1} is a relation from B to A and is defined by R^{-1} = {(b, a) : (a, b) ∈ R}

Equivalence Classes of an Equivalence Relation

Let R be equivalence relation in A (≠ Φ). Let a ∈ A.

Then, the equivalence class of a denoted by [a] or {a} is defined as the set of all those points of A which are related to a under the relation R.

Composition of Relation

Let R and S be two relations from sets A to B and B to C respectively, then we can define Relation SoR from A to C such that (a, c) ∈ So R ⇔ ∃ b ∈ B such that (a, b) ∈ R and (b, c) ∈ S.

This relation SoR is called the composition of R and S.

(I) $ROS \neq SOR$

(ii) ($(ROS)^{-1} = S^{-1}OR^{-1}$

Known as reversal rule.

Congruence modulo m

Let m be an arbitrary but fixed integer. Two integers a and b are said to be congruence modulo

M, if a – b is divisible by m and we write a ≡ b (mod m).

i.e., a ≡ b (mod m) ⇔ a – b is divisible

Important Results on Relation

1. If R and S are two equivalence relations on a set A, then R ∩ S is also on ‘equivalence

Relation on A.

2. The union of two equivalence relations on a set is not necessarily an equivalence relation

On the set.

3. If R is an equivalence relation on a set A, then R^{-1}

is also an equivalence relation on A.

4. If a set A has n elements, then number of reflexive relations from A to A is $2^{n^2} - n$

5. Let A and B be two non-empty finite sets consisting of m and n elements, respectively.

Then, A x B consists of mn ordered pairs. So, total number of relations from A to B is

2^{nm}

Binary Operations

1. Closure Property

An operation * on a non-empty set S is said to satisfy the closure ‘property, if

a ∈ S, b ∈ S ⇒ a * b ∈ S, ∀ a, b ∈ S

Also, in this case we say that S is closed for *.

An operation * on a non-empty set S, satisfying the closure property is known as a binary

Operation.

or

Let S be a non-empty set. A function from S x S to S is called a binary operation on S i.e., f:

S x S → S is a binary operation on set S.

Properties

Generally binary operations are represented by the symbols *, + … etc., instead of

Letters figure etc.

Addition is a binary operation on each one of the sets N, Z, Q, R and C of natural

Numbers, integers, rationales, real and complex numbers, respectively. While addition on

The set S of all irrationals is not a binary operation.

Multiplication is a binary operation on each one of the sets N, Z, Q, R and C of natural

Numbers, integers, rationales, real and complex numbers, respectively. While

Multiplication on the set S of all irrationals is not a binary operation.

Subtraction is a binary operation on each one of the sets Z, Q, R and C of integers,

Rationales, real and complex numbers, respectively. While subtraction on the set of

Natural numbers is not a binary operation.

Let S be a non-empty set and P(S) be its power set. Then, the union and intersection on

P(S) is a binary operation.

Division is not a binary operation on any of the sets N, Z, Q, R and C. However, it is not a

binary operation on the sets of all non-zero rational (real or complex) numbers.

Exponential operation (a, b) → a, b

Is a binary operation on set N of natural numbers

While it is not a binary operation on set Z of integers.

Types of Binary Operations

(i) Associative Law A binary operation * on a non-empty set S is said to be associative, if (a * b) * c

= a * (b * c), ∀ a, b, c ∈ S.

Let R be the set of real numbers, then addition and multiplication on R satisfies the associative

Law.

(ii) Commutative Law A binary operation * on a non-empty set S is said to be commutative, if a * b = b * a, ∀ a, b ∈ S.

Addition and multiplication are commutative binary operations on Z but subtraction not a Commutative binary operation, since

2 — 3 ≠ 3— 2.

Union and intersection are commutative binary operations on the power P(S) of all subsets of set S. But difference of sets is not a commutative binary operation on P(S).

(iii) Distributive Law Let * and o be two binary operations on a non-empty sets. We say that * Is distributed over o., if

a * (b o c)= (a * b) o (a * c), ∀ a, b, c ∈ S also called (left distribution) and (b o c) * a = (b * a) o (c * a), ∀ a, b, c ∈ S also called (right distribution).

Let R be the set of all real numbers, then multiplication distributes addition on R.

Since, a.(b + c) = a .b + a .c,∀ a, b, c ∈ R.

(iv) Identity Element Let * be a binary operation on a non-empty set S. An element e a S, if it Exist such that

a * e = e * a = a, ∀ a ∈ S.

Is called an identity elements of S, with respect to *.

For addition on R, zero is the identity elements in R.

Since, a + 0 = 0 + a = a, ∀ a

For multiplication on R, 1 is the identity element in R.

Since, a x 1 =1 x a = a,∀ a ∈ R

Let P (S) be the power set of a non-empty set S. Then, Φ is the identity element for union on P (S) As

A ∪ Φ =Φ ∪ A = A, ∀ A ∈ P(S)

Also, S is the identity element for intersection on P(S).

Since, A ∩ S=A ∩ S=A, ∀ A ∈ P(S).

For addition on N the identity element does not exist. But for multiplication on N the idenitity .

INVERSE of element

Let * be a binary operation on a non-empty set 'S' and let 'e' be the

Identity element.

Let a ∈ S. we say that a^{-1}

is invertible, if there exists an element b ∈ S such that a * b = b * a =

e

Also, in this case, b is called the inverse of a and we write, a^{-1}

= b

Addition on N has no identity element and accordingly N has no invertible element.

Multiplication on N has 1 as the identity element and no element other than 1 is invertible.

Let S be a finite set containing n elements. Then, the total number of binary operations on S Is N^2

Let S be a finite set containing n elements. Then, the total number of commutative binary

Operation on S is n [n (n+1)/2].